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An optimization process of the wheel profile of tramway vehicles

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Abstract

This paper proposes, using mathematical models, an optimization process of the wheel profile of a tramway vehicle in order to ensure the operational requirements of the network (maximum speeds of 90 km/h on straight paths, negotiation of alignment curvature sections with very small radii (25 -50m)). The research covers tramway vehicles equipped with two different bogie technologies: a) bogies with independently rotating wheels, b) bogies of “mixed” behaviour. The results have evidenced that a) the choice of an optimal wheel profile with the help of mathematical models is theoretically possible b)the optimal wheel profile differs depending on the technology of the bogies

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Keywords: equivalent conicity ; tramway wheels ; tramway bogies; lateral stability of railway vehicles; optimal railway wheel profile; independently rotating wheels.

1. Introduction

This paper proposes, using mathematical models, an optimization process of the wheel profile of a tramway vehicle in order to ensure the operational requirements of the network (maximum speeds of 90 km/h on straight paths, negotiation of alignment curvature sections with very small radii (25 -50m)), with safety, dynamic comfort and rolling quality.

The mathematical models used, allow, for given constructional data characteristics of the vehicle and the track:

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- In straight sections of track, the study of the lateral stability of the vehicle and the calculation of its critical speed (V_{cr}) (Joly and Pyrgidis, 1996; Pyrgidis, 1990)

- In curves, the study of the quasi-static behaviour of the vehicle and specifically (Pyrgidis, 1990 ; Joly and Pyrgidis, 1990) :

  - ✓ The calculation of the lateral displacements (y_1, y_2) and the yaw angles (a_1, a_2) of the two axles of the front bogie of the vehicle
  - ✓ The checking of the rolling conditions of the two wheel-sets of the front bogie of the vehicle (pure rolling, motion with creepage, motion with slip, motion with wheel flange – inner rail side contact)
  - ✓ The calculation, in the case of wheel flange - rail contact, of the guidance forces $F_{ij,}$ applied by the wheels on the rails.
  - ✓ The calculation, in all the rolling cases, of the forces acting on the wheel-sets (creep, gravitational, guidance forces)
  - ✓ In case of motion without wheel flange – rail contact, the evaluation of the total consumed power P on the wheel – rail surface (wear index,all the wheels of the front bogie).

The equivalent conicity is a dimensionless figure and expresses the difference of rolling radii r_1 and r_2 of the two wheels of a specific railway axle, for a given transversal displacement “y” of the axle, considering a specific pose for the track panel ($\tan\gamma_e = \gamma_e = (r_1 - r_2)/2y$) (UIC,2004).Taking into account that the initial wheel tread profile is longitudinally variable (varying conicity), the equivalent conicity is different for a different transversal displacement of the axle from the position of initial equilibrium (axle centered on track).

In order to simulate the wheel-rail contact geometry, the real wheel tread profile is taken into consideration (Shevtsov and all, 2003 ; Pyrgidis and Bousmalis,2010). In the mathematical models used in this work, the wheel profile and more generally the wheel – rail contact geometry are expressed through the value of the equivalent conicity γ_e ($\gamma_e = f(y)$).The equivalent conicity enters, in the mathematic expressions of the creep and gravitational forces applied on the elliptic wheel – rail contact surface and consequently affects the transversal behaviour of railway vehicles.

The research covers tramway vehicles equipped with two different bogie technologies:

- bogies with independently rotating wheels (Pyrgidis,1990 ;Joly and Pyrgidis ,1996)
- bogies of “mixed” behaviour (they operate as conventional bogies in straight path and as bogies with independently rotating wheels in curves) (Pyrgidis,2004)

The whole approach can be applied in railway networks of a different functionality (high speed networks, conventional speed networks, metro networks) changing the constructional characteristics of the bogies (e.g. wheelbase, stiffness of the springs of the suspensions) or/and of the track (e.g. inclination of the rails as to the sleepers)

2. Description of the problem

In the case where the vehicle is equipped with conventional bogies, as evidenced both by the theory (analytical expressions of the creep forces, mathematical models) and by practice, small values of equivalent conicity allow high speeds in straight path. Quite the opposite, in curves, great values of equivalent conicity facilitate the bogie inscription. Based on the above, an "intelligent" wheel profile would constitute a very good solution in the effort to accommodate the forced coexistence of straight / curved sections in a track alignment. For small values of transversal displacement of the axle, (straight path where the transversal displacements are small), such a wheel profile would ensure small values of equivalent conicity, while for great values of transversal displacement (curvatures of track where due to the track geometry, the axles are subject to great transversal displacements), it would ensure great values of equivalent conicity. However, the presence of an "intelligent" wheel profile, such as described here above, takes on its full meaning when the straight track sections are in a very good condition. It is obvious that if the longitudinal defects of the track and the gauge deviation defects are significant, then the transversal displacement of the axle is greater and consequently, a wheel profile that has a great equivalent conicity for great transversal displacements "y" will have a burdening influence on the axle hunting movement (Pyrgidis and Bousmalis, 2010)

In the case where the vehicle is equipped with bogies with independently rotating wheels, the great value of the equivalent conicity facilitates the vehicle movement both in straight path and in curvatures of track. With this technology, the horizontal creep forces become null, the hunting of the axle vanishes and the axle centering on the track is only secured by the gravitational force which, in turn, is proportional to the equivalent conicity (Joly and Pyrgidis, 1990 ; Esveld,2001). Based on the above, an "intelligent" wheel profile should ensure great values of equivalent conicity for small values of transversal displacements of the axle and even greater values of equivalent conicity for great values of transversal displacements.

Based on the above idea, the aim of the paper is the creation of the appropriate theoretical background for optimizing the wheel profile of tramway vehicles equipped with bogies with independently rotating wheels and with bogies of mixed behaviour

3. Description of the reference system track / vehicle

3.1 Construction characteristics of the track and the vehicle

Table 1 shows the symbols adopted in the present paper as well as the values for the track and rolling stock constructional characteristics that remain constant throughout the models processing on the computer.

3.2 Description of the technologies of the bogies

The vehicle considered in the study is made up of the car body and two two-wheel set bogies.(Pyrgidis,1990).The bogies frame is connected to the body and to the axles with the help of flexible couplings (e.g. helical springs) and dampers offering the vehicle two levels of suspension:

- primary suspension (bogie - axle) with four springs per bogie in total
- secondary suspension (bogie - body) with two springs per bogie in total

With respect to bogies, two different technologies were considered:

Bogies with independently rotating wheels

In this technology broadly used in modern tramway vehicles each bogie bears four wheels running at different angular velocities (freely). With this technology that can be implemented with or without axles, the hunting effect of the axles is avoided, as the horizontal creep forces vanish.

Bogies of "mixed" behavior

These motor bogies are equipped with a differential device that allows the two wheels of each axle to rotate, while cornering, at different speeds continuing to transmit the full traction torque on the track. When the vehicle runs on straight sections or wide radius curves, the device stiffly re-connects the two wheels with respect to each other. (The axle-body and wheels rotating with the same angular velocity)

Table 1: Symbols – Constructional characteristics of the track and the reference vehicle

$\overline{M}$	: Mass of car - body	= 19,000kg
M	: Mass of bogie frame	= 1,200kg
m	: Mass of wheel set	= 900kg
$\widehat{m}$	: Mass of axle boxes	= 100kg
m_r	: Mass of a pair of wheels	= 300kg
$\overline{K}_x$	: Longitudinal rigidity of the secondary suspension	= 2.00 x 10^5N/m
$\overline{K}_y$	: Transversal rigidity of the secondary suspension	= 2.00 x 10^5N/m
$\overline{K}_z$	: Vertical rigidity of the secondary suspension	= 5.30 x 10^5N/m
$\overline{K}_o$	: Longitudinal rigidity of the anti-yaw devices	= 0.
K_x	: Longitudinal rigidity of the primary suspension	= 2.00 x 10^6N/m
K_y	: Transversal rigidity of the primary suspension	= 5 x 10^5N/m
K_z	: Vertical rigidity of the primary suspension	= 0.975 x10^6N/m
$\overline{C}_y$	: Viscose transversal damping of the secondary suspension	= 0.35 x 10^5N/m/sec
$\overline{C}_z$	: Viscose vertical damping of the secondary suspension	= 0.15 x 10^5N/m/sec
C_z	: Viscose vertical damping of the primary suspension	= 0.108 x 10^5N/m/sec
2α	: Bogie wheel base	= 1.80m
$2\overline{A}$	: Distance of pivot of bogies	= 14.00m
r_o	: Wheel rolling radius when the axle is centred on the track	= 0.30m
R'	: Radius of curvature of the rail head at the contact point rail - wheel	= 0.30m
R	: Radius of curvature of the wheel profile at the contact point rail – wheel	varies
γ_o	: Inclination of the tangent plane at the contact point between rail and wheel when the wheel set is in central position	= 0.025
γ_{nc}	: non compensated centrifugal acceleration	= 0.
N	: Load per wheel	= 31,750N
γ_e	: Wheel equivalent conicity	varies
σ	: Clearance between the wheel flange and the rail	= 7.50mm
R_c	: Curve radii	25 to 50m
μ	: Coefficient of friction of Coulomb in curves	= 0.60
$2e_o$	: Distance between the running surfaces of the right and the left wheel in central position	= 1.50 m
$2\overline{d}$, 2d	: Transversal distances between springs or dampers of the secondary and primary suspension	= 2.0m
2pl	: Diameter of wheel set	= 0.080m
U_{max}	: Maximum normal cant of the track	= 150 mm
V	: Running speed of the vehicle	
V_{cr}	: Critical speed of the vehicle	
i,j	: Indicators related to the axles (i=1,2) and to the wheels (j=1,2)	

F_{ij} : Guidance force
y_i : Transversal displacement of the axles
a_i : Axle yaw angles
P : Total consumed power on the wheel – rail interface (1 bogie, 4 wheels)

3.3 Wheel treads profiles

In order to apply the proposed optimization process of the wheel treads profile, the following profiles are considered:

- In the case of tramway vehicles equipped with bogies of mixed behaviour, the 4 profiles given in Figure 1. One of the above profiles corresponds to a specific profile (tram Sirio of Athens). The other profiles are hypothetical.
- In the case of tramway vehicles equipped with bogies with independently rotating wheels (in straight paths and in curves), the 4 profiles given in Figure 2. All the profiles are hypothetical. The common trait these profiles share is the upwards trend of γ_e from the beginning, as in the independently rotating wheels, great value of equivalent conicity does not affect the stability in straight path since the hunting effect is eliminated.

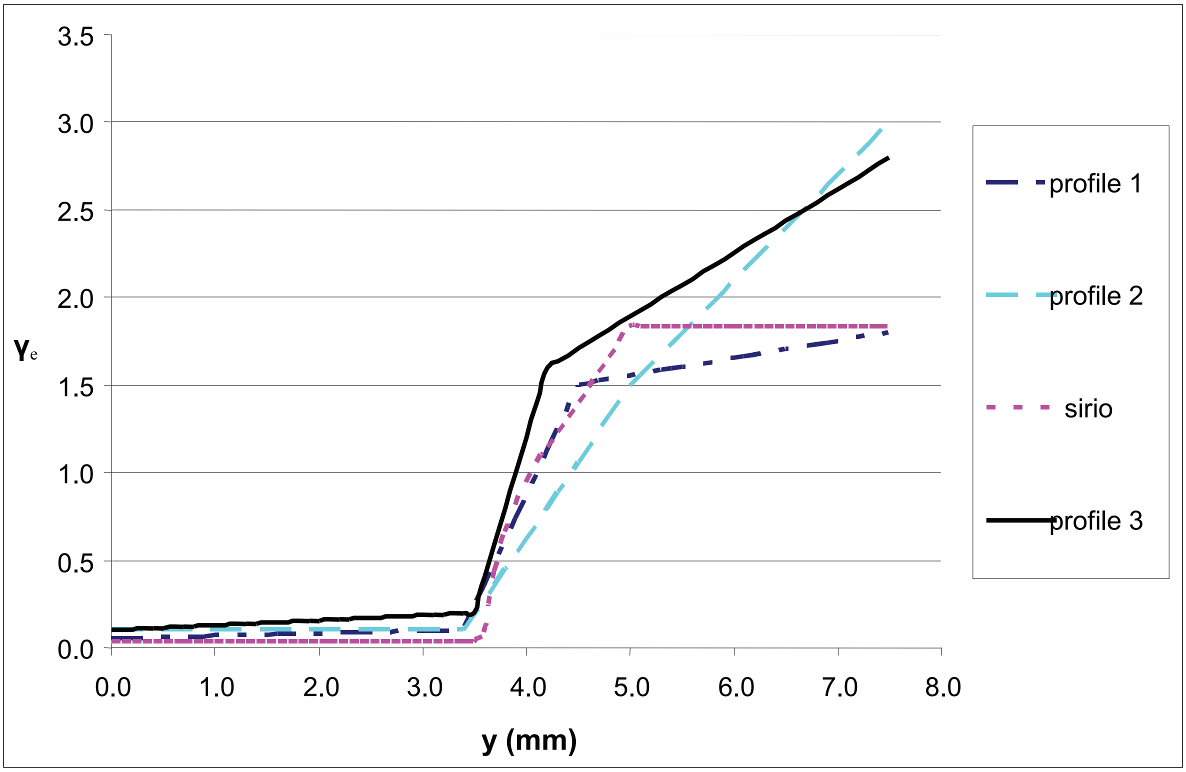

Figure 1: Tramway bogies of mixed behaviour. Indicative profiles adopted for the application of the proposed process

At this point, it should be clarified that it is more correct to define first the constructional equation of the profiles and then to obtain the function $\gamma_e = f(y)$ of figures 1 and 2 after an appropriate mathematical processing (linear regression r_1-r_2 = f(y)) (UIC, 2004). However, this approach was not adopted, given that the objective is not the choice of the optimal wheel profile but the creation of a procedure of optimization of the profile of tramway vehicle wheels.

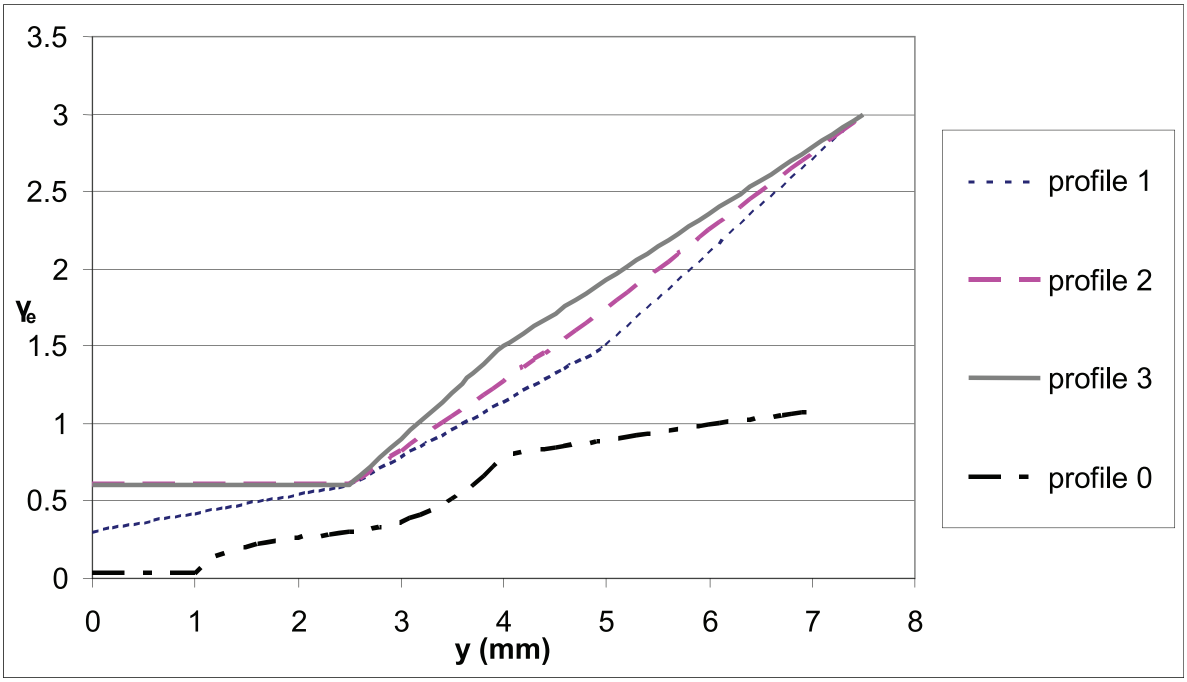

Figure 2: Tramway bogies with independently rotating wheels. Indicative profiles adopted for the application of the proposed process

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4. Vehicles with bogies of mixed behaviour

4.1 Study of the transversal stability in straight path

The study of the transversal behaviour of the vehicle in straight path is carried out with differential equation systems, resulting from the application of Lagrange formula, which simulates the behaviour of a vehicle equipped with conventional bogies at the level of the front bogie (Pyrgidis, 1990). With these mathematical models and with the appropriate software that was configured, it is possible to calculate the critical speed, for a specific equivalent conicity and for given constructional characteristics of the vehicle

In straight path it is considered that:

- the vehicle runs at a constant speed on track without gradient and track defects
- the wheel set guiding during the motion is only secured by the combined action of the wheel equivalent conicity and of the creep forces (no flange contact appears)
- the creep forces are expressed according to the linear theory of Kalker (Kalker,1967)
- the equivalent conicity of the wheels has a constant value (relation 1, Joly and Pyrgidis, 1996)

$$\gamma_e = \frac{R\gamma_o}{R - R'} \qquad (1)$$

This relation considers a circular wheel rim profile and takes into account the wear on the wheel and rail-rolling surface that is a major factor for the stability and guidance of the vehicles.

Specifically it was considered that the axle transversal displacement will not exceed ± 3.5 mm. Based on this assumption and taking in consideration the relations γ_e=f(y) presented in the diagrams of Figure 1, the equivalent conicity average values mentioned in Table 2 (column 2) derive for each separate profile.

The criterion governing the vehicle performances is:

1. The critical speed V_{cr} of the vehicle. A greater critical speed entails a better profile in straight path.

4.2 Study of the quasi - static behaviour in curves

The study of the transversal behaviour of the vehicle in the curvature sections of the track is carried out with the aid of linear mathematical models that simulate the transversal quasi-static behaviour of a railway vehicle equipped with independently rotating wheels (Pyrgidis, 2004).

In curves:

- for the calculation of the expressions of creep forces, a non-linear theory was adopted (Johnson and Vermeulen ,1964)
- spin effect was ignored
- a single contact point is considered between rail and wheel
- for the study of the contact geometry, the real tread profile has been considered (only for the two wheels of the front axle). The algorithm resolves the non linear system that simulates the transversal behaviour of the vehicle using Bolzano's theorem, progressively increasing the transversal displacement of the guiding wheel set and assuming initially null values. These values simultaneously enter into the equations γ_e = f(y) of the wheel profile and define respective equivalent conicity values. The above equations of the profiles will have to be satisfied by the system resolution results.

The criteria governing the vehicle performances in curves are:

2. The axle rolling conditions. The best condition is the one of pure rolling , followed by the wheel creepage without wheel flange - rail contact , then by the creepage with contact while the worst condition is the one with slip and wheel flange - rail contact

3. When there is a wheel flange - rail contact, the value of the guiding force F_{ij}. The smaller the value of F_{ij} the better the profile.

4. When a wheel flange - rail contact does not take place a) The value of the consumed power P at the level of the 4 wheels of the bogie. The smaller the value of P the better the profile in the curvatures of track. b) the lateral displacement y_1 of the outer wheel of the front axle of the bogie. The smaller the value y_1 the better the profile in curvatures of track.

To evaluate the total consumed power P at wheel – rail contact level the wear index proposed by Elkins and Eickhoff, 1982, is used

4.3 Application results

The optimal profile is the one that meets comparatively better the above 4 criteria simultaneously. (criterion 1 in straight path and criteria 2,3,and 4 in curves)

Table 2: Tramway bogies of mixed behaviour - Performance of vehicles in straight path and curvatures of track – Different wheel –rail contact geometry

PROFILE	γ_e V_{cr}	R_c=25m V= 17.83 km/h	R_c=50m V=25.21 km/h
(1)	(2)	(3)	(4)
Sirio	0.03 281.38km/h	y_1=+5.920 mm , y_2=-0.731 mm F_{ij}= 0 t P = 22,071.8KW γ_e=1.803 (slip of both axles)	y_1=+ 4.802 mm, y_2=+1.049 mm F_{ij}= 0 t P =15,699.5 KW γ_e=1.617 (slip of the front axle only)
1	0.075 169.38km/h	y_1=+5.051 mm , y_2=-0.628 mm F_{ij}= 0 t P = 21,900.4KW γ_e=1.716 (slip of both axles)	y_1=+ 4.683 mm, y_2=+1.023 mm F_{ij}= 0 t P =15,687.9 KW γ_e=1.666 (slip of the front axle only)
2	0.10 148.25km/h	y_1=+5.700 mm , y_2=-0.710 mm F_{ij}= 0 t P = 22,023.6KW γ_e=1.884 (slip of both axles)	y_1=+ 5.100 mm, y_2=+1.120 mm F_{ij}= 0 t P =15,757.2 KW γ_e=1.518 (slip of the front axle only)
3	0.15 125.60km/h	y_1=+5.363 mm , y_2=-.665 mm F_{ij}= 0 t P = 21,963.2KW γ_e=1.996 (slip of both axles)	y_1=+ 4.541 mm, y_2=+0.991 mm F_{ij}= 0 t P =15,657.0 KW γ_e=1.698 (slip of the front axle only)

Table 2 features for the four wheel profiles examined:

a) In straight path (column 2):
- The corresponding equivalent conicity γ_e
- The critical speed V_{cr} of the vehicle

b) In curvatures of track (columns 3 and 4):
- For horizontal alignment radii 25 and 50m (columns 3 and 4 respectively)
  - ✓ The transversal displacement y_1 and y_2 of the two axles of the vehicle front bogie
  - ✓ The corresponding equivalent conicity γ_e
  - ✓ The guidance force F_{ij} if applicable(case of wheel flange - rail contact)

- ✓ The total consumed power P at the wheel - rail contact level (4 wheels of the front bogie) in the case where rolling takes place without a wheel flange - rail contact
- ✓ The transit speed
- ✓ The rolling conditions

When we examine the results of table 2, we can derive the following:

With regard to the movement of vehicles in straight paths, profile Sirio presents the optimum behavior. This profile allows the greatest theoretical critical velocity (281.38 km/h , hence a security factor of 3.12 with regards to the greatest allowed velocity in a tramway network (90 km/h)).

It must be noted though, that in case of track defect (as far as horizontal alignment is concerned) or deviation in the track gauge, these factors can produce a transversal displacement $y \geq 3.5$mm. In that case the vehicle faces serious stability problems, since e.g. for the profile Sirio on y =4mm we have γ_e=1.0, which with a K_x=2x10^6 N/m and K_y=5x 10^5 N/m gives V_{cr}= 93.10km/h, while for even greater displacements, like γ_e=1.5, for the same stiffness values we calculate V_{cr} = 84.49 km/h.

The rest of the three examined profiles cover the operational requirements, however offer much tighter safety margins with regards to velocity (eg for profile 3 the security factor is 1.38).

With regard to the negotiation of bogies in horizontal alignment curvatures of track, profile 1 prevails as it satisfies comparatively better, for the two radii that were examined, all three criteria set in paragraph 4.2 for the movement in curves.

When examining the behaviour of bogies in both straight paths and curvatures of track, it appears that the optimum profile would be a combination of profile Sirio and profile 1 (figure 3), that is a profile which for $y < y_{crit}$ follows the pattern of profile Sirio and for $y > y_{crit}$ follows the pattern of profile 1. The performance results of this new suggested profile are given in Table 3

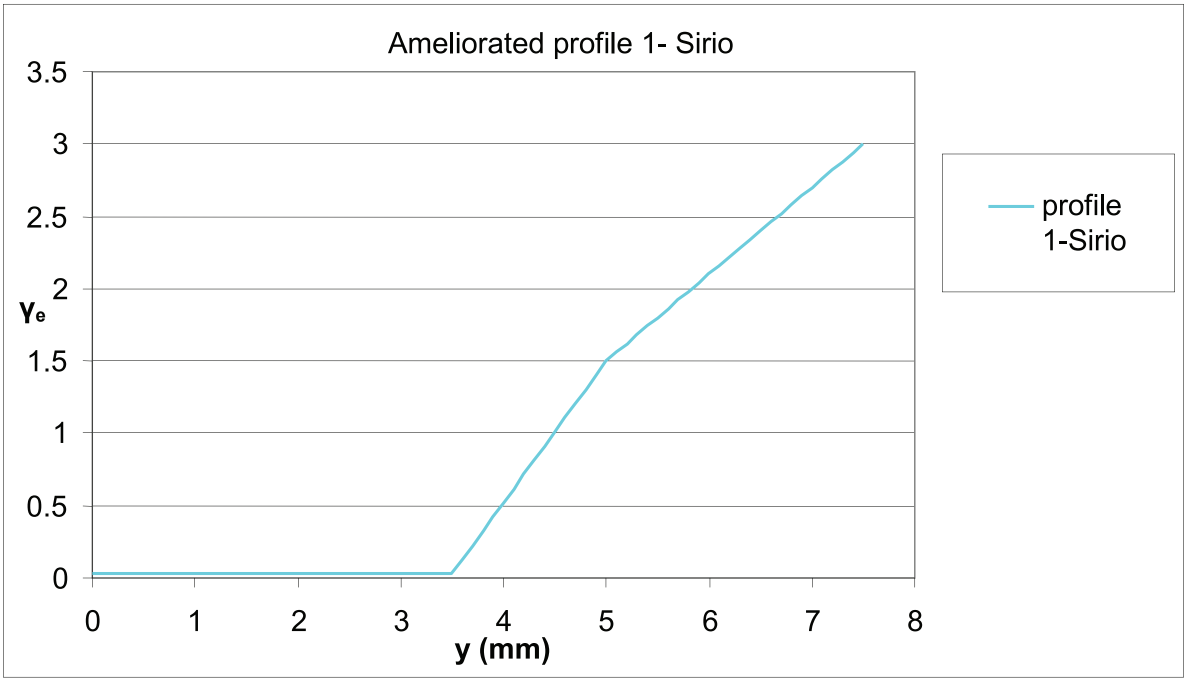

Figure 3: Tramway bogies with independently rotating wheels. Proposed ameliorated profile

Table 3: Tramway bogies of mixed behaviour - Performance of vehicles in straight path and curvatures of track in the case of the proposed ameliorated profile

γ_e V_{cr}	R_c=25m V= 17.83 km/h	R_c=50m V=25.21 km/h
0.030 281.38km/h	y_1=+5.700 mm , y_2=-0.705 mm F_{ij}= 0 t P = 22,023.6KW	y_1=+ 5.104 mm, y_2=+1.117 mm F_{ij}= 0 t P =15,757.2 KW

5. Vehicles with bogies with independently rotating wheels

5.1 Study of the transversal stability in straight path

It was considered that the axle transversal displacement will not exceed ± 3.5 mm. Based on this assumption and taking in consideration the relations γ_e=f(y) presented in the diagram of Figure 2, the equivalent conicity average values mentioned in Table 4 (column 2) derive for each separate profile.

The bogies with independently rotating wheels theoretically outclass in straight path, yet, as mentioned before, they are very sensitive to any cause that might provoke a transversal displacement of the axle as the only force that can henceforth centre the axle on the track is the gravity force. (Pyrgidis, 2004).This force is proportional to the displacement ''y'' of the axle and to the wheel equivalent conicity γ_e

The criterion governing the vehicle performances is:

5. The gravitational force in straight path or otherwise the value of the equivalent conicity. The higher the value of γ_e the better the profile.

5.2 Study of the quasi- static behaviour in curves

Exactly the same approach as in the case of bogies of mixed behaviour. The criteria governing the vehicle performances in curves are the same

5.3 Application results

The optimal profile is the one that meets comparatively better the above 4 criteria simultaneously. (criterion number 5 in straight path and criteria 2,3,and 4 in curves)
When we examine the results of table 4, the following derive:

With regard to the movement of vehicles in straight paths, the speed is theoretically infinite for all the examined profiles. However, judging by the value of the equivalent conicity, profile 3 has the best performance since it has the greatest value of γ_e.

With regard to the negotiation of bogies in horizontal alignment curvatures of track profile 3 prevails as it satisfies comparatively better, for the two radii that were examined, all three criteria set in paragraph 5.2 for the movement in curves. Profile 0 is the only case where we have direct contact between the wheel flange and the rail (for R_c=25m) However the relevant forces remain low and acceptable.
Table 4: Tramway bogies with independently rotating wheels - Performance of vehicles in straight path and curvatures of track – Different wheel –rail contact geometry

PROFILE	γ_e V_{cr}	R_c=25m V= 17.83 km/h	R_c=50m V=25.21 km/h
(1)	(2)	(3)	(4)
0	0.266 infinite	y_1=+7.500 mm , y_2=-1.166 mm F_{11}= 1.866 t γ_e=1.127 (flange contact of the front axle)	y_1=+7.126 mm, y_2=+1.574 mm F_{ij}= 0 t P =16,135.1KW γ_e=1.085 (slip of both axles)
1	0.610 infinite	y_1=+5.700 mm , y_2=-0.705 mm F_{ij}= 0 t P = 22,023.6KW γ_e=1.884 (slip of both axles)	y_1=+ 5.104 mm, y_2=+1.117 mm F_{ij}= 0 t P =15,757.2 KW γ_e=1.518 (slip of the front axle only)
2	0.730 infinite	y_1=+5.485 mm , y_2=-0.680 mm F_{ij}= 0 t P = 21,984.6KW γ_e=1.955 (slip of both axles)	y_1=+ 4.786 mm, y_2=+1.046 mm F_{ij}= 0 t P =15,700.5 KW γ_e=1.615 (slip of the front axle only)
3	0.780 infinite	y_1=+5.295 mm , y_2=-0.657 mm F_{ij}= 0 t	y_1=+ 4.536 mm, y_2=+0.990 mm F_{ij}= 0 t

		P = 21,950.0KW γ_e=2.023 (slip of both axles)	P =15,655.4 KW γ_e=1.701 (slip of the front axle only)

When examining the behaviour of bogies in both straight paths and curvatures of track, it appears that profile 3 has the best performance, while profile 1 and profile 2 produce slightly worse results. Profile 0 is judged as unacceptable.

6. Conclusions

This paper proposes, using mathematical models, an optimization process of the wheel profile of a tramway vehicle in order to ensure the operational requirements of the network (speeds 70-90 km/h on straight paths, negotiation of alignment curvature sections with very small radii (25 -50m)), with safety, dynamic comfort and rolling quality .
The research covers tramway vehicles equipped with two different bogie technologies: a) bogies with independently rotating wheels, b) bogies of "mixed" behaviour. For the study of the contact geometry in curves, the real tread profile has been considered

The results have evidenced that in the case of tramway vehicles:

- The optimal wheel profile differs depending on the technology of the bogies
- The optimal wheel profile differs depending also on the condition of the track (track defects)

In any case a first theoretical approach to obtain the optimal wheel profile using mathematical models is always a significant help to the manufacturer. Testing in real operating conditions represents the last step for the acceptance of a product.

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